Hw 6 _ 2050B _ Oct 2020

1.* Let $f : \mathbb{R} \to \mathbb{R}$ be additive :

$$f(x+y) = f(x) + f(y) \quad \forall\, x, y \in \mathbb{R}.$$

Show that

$$f(0) = 0 \;,$$

$$f(-x) = -f(x), \quad \forall\, x \in \mathbb{R} \;;$$

$$f(nx) = n f(x) \quad \forall\, n \in \mathbb{Z},\ x \in \mathbb{R} \;;$$

$$f\left(\frac{x}{m}\right) = \frac{f(x)}{m} \quad \forall\, m \in \mathbb{N} \;;$$

$$f(rx) = r f(x) \quad \forall\, r \in \mathbb{Q} \ \&\ x \in \mathbb{R}$$

and that

$\lim_{x \to x_0} f(x)$ exists in $\mathbb{R}$ for some $x_0 \in \mathbb{R}$ iff $\lim_{x \to c} f(x)$ exists in $\mathbb{R}$ for any $c \in \mathbb{R}$ (what then $\lim_{x \to 0} f(x)$ is ?).

Show further that (assuming $\lim_{x \to x_0} f(x)$ exists in $\mathbb{R}$ for some $x_0 \in \mathbb{R}$), with $K := f(1)$,

$$f(x) = Kx \quad \forall\, x \in \mathbb{R}.$$

2. The last four questions (Q13-16 of 3rd Ed = Q14-17 of 4th Ed. of Bartle-Sherbert.

3.* Use ε-δ definition to check that

$$\lim_{x \to -1} \frac{x+5}{2x+3} = 4 \;;$$

$\lim_{x \to 0} (x + \operatorname{sgn}(x))$ & $\lim_{x \to 0} \left(\sin \frac{1}{x^2}\right)$ not exist in $\mathbb{R}$.

4. Q1, Q3, Q8 – Q11 and the last question (Q14 of 3rd Ed. = Q15 of 4th Ed.) ~~of~~ of § 4.2 in Bartle-Sherbert.)